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Multiphoton entanglement and interferometry

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Multiphoton entanglement is the basis of many quantum communication schemes, quantum cryptographic protocols, and fundamental tests of quantum theory. Spontaneous parametric down-conversion is the most effective source for polarization entangled photon pairs. Here we show, that a class of entangled 4-photon states can be directly created by parametric down-conversion. These states exhibit perfect quantum correlations and a high robustness of entanglement against photon loss. Therefore these states are well suited for new types of quantum communication.

1 Introduction

Entangled states are key elements in the field of quantum information processing. The experimental preparation, manipulation and detection of multi-photon entangled states is of great interest for implementations of quantum communication schemes, quantum cryptographic protocols, and for fundamental tests of quantum theory. Parametric down conversion has been proven to be the best source of entangled photon pairs so far in an ever increasing number of experiments on the foundations of quantum mechanics [1] and in the new field of quantum communication. Experimental realizations of concepts like entanglement based quantum cryptography [2], quantum teleportation [3] and its variations [4] demonstrated the usability of this source. New proposals for quantum communication schemes [5–7] and, of course, for improved tests of local hidden variable theories initiated the quest for entangled multi-photon states. Interference of photons generated by independent down conversion processes enabled the first demonstration of a 3-photon Greenberger-Horne-Zeilinger (GHZ)-argument [8] and, quite recently, even the observation of a 4-photon GHZ-state [9].

Instead of sophisticated but fragile interferometric set-ups, we utilize bosonic interference in a double-pair emission process. This effect causes strong correlations between measurement results of the 4 photons and renders type-II down conversion a valuable tool for new multi party quantum communication schemes. The analysis of the entanglement inherent in the four photon emission leads us to a new form of inequality distinguishing local hidden variable theories from quantum mechanics, and demonstrates its potentiality for experiments on the foundations of quantum mechanics. In this contribution we present new schemes to prepare 4- and 3-photon entangled states (Sect. 2) and present first experimental results (Sect. 3). Detailed studies of quantum correlations and of the robustness of entanglement are presented in Sect. 4 and 5. In conclusion (Sect. 6) we address possible applications for multiparty quantum communication.

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2 Entangled multi-photon state preparation

In type-II parametric down conversion [10] multiple emission events during a single pump pulse lead to the following state

$$C \exp\left(-i\alpha(a_V^* b_H^* + a_H^* b_V^*)\right)|0\rangle, \tag{1}$$

where C is a normalization constant, α is proportional to the pulse amplitude, and where a_V^* is the creation operator of a photon with vertical polarization in mode a, etc. The creation of one pair of entangled photons corresponds to the first order term of expansion on α of Eq. (1).

$$(a_H^* b_V^* + a_V^* b_H^*)|0\rangle. \tag{2}$$

This 2-photon entangled state has become a routine in the laboratory and it has been used for tests of non-locality in quantum mechanics and in several experimental realizations of two party communication protocols [2, 3].

2.1 Entangled four-photon state

The second order term corresponds to the emission of 4 photons and it is proportional to

$$(a_H^* b_V^* + a_V^* b_H^*)^2|0\rangle. \tag{3}$$

The particle interpretation of this term can be obtained by its expansion

$$(a_H^{*2} b_V^{*2} + a_V^{*2} b_H^{*2} + 2a_V^* a_H^* b_V^* b_H^*)|0\rangle, \tag{4}$$

and is given by the following superposition of photon number states

$$|2H_a, 2V_b\rangle + |2V_a, 2H_b\rangle + |1H_a, 1V_a, 1H_b, 1V_b\rangle, \tag{5}$$

where e.g. $2H_a$ means 2 horizontally polarized photons in the beam a and $2V_b$ means 2 vertically polarized photons in the beam b.

One should stress here that this type of description is valid only for the down conversion emissions, which are detected behind filters endowed with a frequency band, which is narrower than that of the pumping fields [11]. If a wide band down-conversion is accepted then such a state is effective only if counts at the detectors are treated as coincidences, when they occur within time windows narrower than the inverse of the bandwidth of the radiation [12]. If such conditions are not met, then the four photon events are essentially emissions of two independent, entangled pairs, with the entanglement existing only within each pair.

Let us pass the 4-photon state via two polarization dependent variable beam splitters. At the beam splitter a_H is transformed into $\cos(\theta_H)a_H + \sin(\theta_H)a'_H$ and a_V into $\cos(\theta_V)a_V + \sin(\theta_V)a'_V$, with prime denoting the reflected beam. For simplicity we assume the same splitting ratio for the arm b. One can expand the expression (5), and then extract only those terms that lead to 4-photon coincidence behind the two beam splitters, i.e. only those terms for which there is one photon in each of the output ports. The resulting component of the full state is given by

$$\begin{aligned}
&\sin(2\theta_H)\sin(2\theta_V)\left[|HHVV\rangle_{aa'bb'} + |VVHH\rangle_{aa'bb'}\right] \\
&+ \left[\cos(\theta_H)\sin(\theta_V)|HV\rangle_{aa'} + \cos(\theta_V)\sin(\theta_H)|VH\rangle_{aa'}\right] \\
&\times \left[\cos(\theta_H)\sin(\theta_V)|HV\rangle_{bb'} + \cos(\theta_V)\sin(\theta_H)|VH\rangle_{bb'}\right],
\end{aligned} \tag{6}$$

where we now use the notation of the first quantization with $|H\rangle_a$ describing an horizontally polarized photon in mode a and etc. The first term represents a 4-photon GHZ state, whereas the second one is a product of 2-photon entangled states. This shows that from down-conversion, we are able to produce directly a class of entangled four-photon states without the need for fragile, interferometric setups.

For the particular case of $\theta_H = \theta_V = \pi/4$, we obtain after normalization and taking into account the phase shift in the reflected arm on the beam splitters:

$$|\psi\rangle^4_{aa'bb'} = \sqrt{2/3}|\,\mathrm{GHZ}\rangle_{aa'bb'} + \sqrt{1/3}|\,\mathrm{EPR}\rangle_{aa'}|\,\mathrm{EPR}\rangle_{bb'}\,. \tag{7}$$

For additional simplicity of the presentation we also rotate the polarizations in the beams a and a' by $90°$. Thus now our initial state is given by (7) with the GHZ state in its standard form, resulting in the state [13]:

$$\sqrt{1/3}\big(|VVVV\rangle_{aa'bb'} + |HHHH\rangle_{aa'bb'} \\ -\tfrac{1}{2}\left(|HVHV\rangle_{aa'bb'} - |HVVH\rangle_{aa'bb'} - |VHHV\rangle_{aa'bb'} + |VHVH\rangle_{aa'bb'}\right)\big). \tag{8}$$

2.2 Three-photon entangled W state

Dür et al have shown that there are only two classes of genuinely three particle entangled states which are inequivalent under stochastic local quantum operations and classical communications. The first class is represented by the GHZ state and the second by the so-called W state defined as follows [14]:

$$|\,\mathrm{W}\rangle_{abc} = \sqrt{1/3}\,[|HHV\rangle_{abc} + |HVH\rangle_{abc} + |VHH\rangle_{abc}] \tag{9}$$

Recently, the 3-photon GHZ state has been demonstrated experimentally by interferometrically overlapping entangled photon pairs [8]. We present a schematic setup for the W state preparation similar to the GHZ setup. The experimental setup is such that the photons in arm (a) are split by a polarization beam splitter, the V photon is reflected and continues towards a trigger detector D_T. The photons in arm (b) are split by a beam splitter with a $1/3$ splitting ratio, where one output continues towards an analyzer D_3 and the second output continues towards to a beam splitter for an overlap with the photon transmitted from the first polarization beam splitter. Finally, the two outputs are directed to two analyzers D_1 and D_2. The experimental realization of this setup is currently under preparation in our laboratory.

3 Experiment

In the experiment, we used the UV-pulses of a frequency doubled mode-locked Ti:saphire laser with repetition rate of 76 MHz to pump a 2mm thick BBO crystal. The degenerate down-conversion emission at the two characteristic type II crossing points was coupled into single mode fibers to exactly define the emission modes and then filtered with narrowband interference filter ($\Delta\lambda = 3$ nm). After the two $50/50$ beam splitters, we obtain four modes (a,a',b,b') (see Fig. 1). We used quarter and half wave plates to set the analysis orientations. The four photons where detected by single photon Si avalanche photodiodes and analyzed with eight-channel multi-coincidence logic. This specially designed system simultaneously registers any possible coincidence between the eight detectors and thus allows an efficient registration of the 16 relevant four-fold coincidences. Fig. 2 shows the 16 possible four-fold coincidences for detecting the four entangled photons when all four polarization analyzers are oriented along H/V, $+45^o\backslash -45^o$, and left/right polarizations. We see clearly the superposition of a GHZ state and a product of two EPR states.

4 Quantum correlations

Quantum correlations are essential for experiments on the violation of a Bell inequality and for quantum cryptographic protocols. Let us analyze polarization correlation measurements involving all four modes (a,a',b,b'), where the actual observables to be measured are elliptic polarizations at 45^o. Such observables

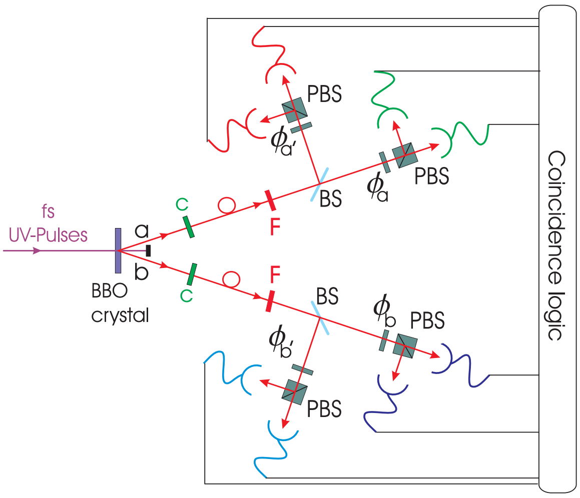

Fig. 1 (online colour at: www.interscience.wiley.com) Experimental setup for the demonstration of four-photon entanglement where C, F, BS, and PBS stand for fiber coupler, filter, non-polarizing beam splitter and polarizing beam splitter respectively. The 4 photons emitted into modes a and b are split by BS). Four polarization analyzers with different settings ($\phi_i(i = a, a', b, b')$) are used.

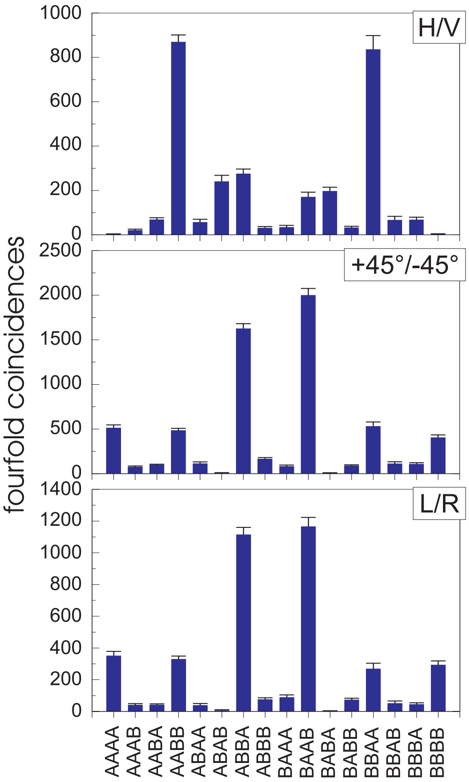

Fig. 2 (online colour at: www.interscience.wiley.com) Fourfold coincidences, for the entangled 4-photon state (Eq. (7)), with different polarization settings where A and B stand for (H, $+45^o$, left) and (V, -45^o, right) polarization respectively.

are of dichotomic nature, i.e. endowed with two valued spectrum $k = +1, -1$, and are defined for each spatial propagation mode $x = a, a', b, b'$ by their eigenstates

$$\sqrt{1/2}|V\rangle_x + k\mathrm{e}^{-i\phi_x}\sqrt{1/2}|H\rangle_x = |k, \phi_x\rangle. \tag{10}$$

The probability amplitudes for the results $k, l, m, n = \pm 1$ at the detector stations in the beams a, a', b, b', under local phase settings $\phi_a, \phi_{a'}, \phi_b, \phi_{b'}$, respectively, are given by

$$\frac{1}{4\sqrt{3}}\left[1 + klmn\,\mathrm{e}^{i\sum\phi} + \tfrac{1}{2}(k\,\mathrm{e}^{i\phi_a} + l\,\mathrm{e}^{\phi_{a'}})(m\,\mathrm{e}^{i\phi_b} + n\,\mathrm{e}^{i\phi_{b'}})\right], \tag{11}$$

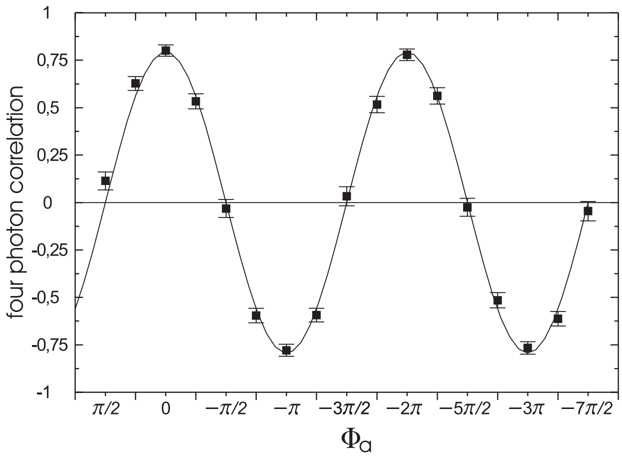

Fig. 3 Quantum correlation function (Eq. (14)) on the angle ϕ_a and the other analyzers are fixed at angle $\phi'_a = \phi_b = \phi'_b = 0$. The data shows a visibility of $V \simeq 75\,\%$.

where the $\sum \phi = \phi_a + \phi'_a - \phi_b - \phi'_b$. Therefore the probability to get a particular set of results (k, l, m, n) is given by

$$P(k,l,m,n|\phi_a,\phi_{a'},\phi_b,\phi_{b'}) = \tfrac{1}{16}\Big[\tfrac{2}{3}(1 + klmn\cos\sum\phi) + \tfrac{1}{3}(1 + kl\cos(\phi_a - \phi_{a'}))(1 + mn\cos(\phi_b - \phi_{b'})) + \tfrac{1}{3}\,\mathrm{Re}\Big((1 + klmn\,\mathrm{e}^{i\sum\phi})(k\,\mathrm{e}^{i\phi_a} + l\,\mathrm{e}^{i\phi_{a'}})(m\,\mathrm{e}^{i\phi_b} + n\,\mathrm{e}^{\phi_{b'}})\Big)\Big]. \qquad (12)$$

The last term is written in the form of a real part of a complex function to shorten the expression. The correlation function is defined as the mean value of the product of the four local results

$$E(\phi_a,\phi_{a'},\phi_b,\phi_{b'}) = \sum_{k,l,m,n=\pm 1} klmn\,P(k,l,m,n|\phi_a,\phi_{a'},\phi_b,\phi_{b'}). \qquad (13)$$

Its explicit form for the considered state (Eq. (7)) is thus

$$E(\phi_a,\phi_{a'},\phi_b,\phi_{b'}) = \tfrac{2}{3}\cos(\phi_a + \phi'_a - \phi_b - \phi'_b) + \tfrac{1}{3}\cos(\phi_a - \phi_{a'})\cos(\phi_b - \phi_{b'}). \qquad (14)$$

The correlation function is a weighted sum of the GHZ correlation function (the first term) and a product of two EPR-Bell correlation functions. We note that our four-photon entangled state violates a generalized Bell inequality (for detail see [13]). Fig. 3 shows the dependence of the correlation function (Eq. (14)) on the angle ϕ_a when the other analyzers are fixed at angle $\phi'_a = \phi_b = \phi'_b = 0$. The data show a visibility of $V \simeq 75\%$

5 Entanglement robustness

Entangled states are subject to decoherence and particle losses due to their interaction with the environment. For the four photon entangled state, we have analyzed two cases. The first case, when two photons are lost from the modes a' and b', and the second, when they are lost from the modes b and b'. Mathematically this corresponds to trace out two qubits and the density matrix of the remaining qubits becomes (with the four photons density matrix defined by $\rho^4_{aa'bb'} = |\psi\rangle^4\langle\psi|_{aa'bb'}$)

$$\rho_{ab} = Tr_{a'b'}[\rho^4_{aa'bb'}] = \tfrac{2}{3}|\psi^+\rangle_{ab}\langle\psi^+| + \tfrac{1}{3}\frac{I_{ab}}{4} \qquad (15)$$

or

$$\rho_{aa'} = Tr_{bb'}[\rho^4_{aa'bb'}] = \tfrac{1}{3}|\psi^-\rangle_{aa'}\langle\psi^-| + \tfrac{1}{3}[|HH\rangle_{aa'}\langle HH| + |VV\rangle_{aa'}\langle VV|] \quad (16)$$

respectively, where I is unit density matrix $I = |HH\rangle\langle HH| + |VV\rangle\langle VV| + |HV\rangle\langle HV| + |VH\rangle\langle VH|$. A strong criterion for entanglement analysis is the Peres-Horodecki criterion, which states that for separable states ρ, the partial transpose of the density matrix ρ must have nonnegative eigenvalues [16]. We apply this criterion to the above reduced density matrices and obtain $3\lambda_i = \{-3/4, 5/4, 5/4, 5/4\}$ and $3\lambda_i = \{1/2, 1/2, 1/2, 3/2\}$ respectively. The first state (Eq. (15)) is partially entangled and there are purification procedures to transform this state to a pure entangled state [17] or one can obtain a state close to an EPR state by means of a filtering measurement [18]. The second state (Eq. (16)) is a separable state and possesses only classical correlations.

6 Conclusions

Quantum entangled states are used as resources for quantum information processing. Perfect correlations are present in our four-photon entangled state, for instance, when $\phi'_a = \phi_b = \phi'_b = 0$, the quantum correlation function becomes simply $\cos(\phi_a)$ and exhibits perfect correlation between the measurement outcomes. Perfect correlations together with the violation of a generalized Bell-inequality are the ingredients for secure multi-party quantum communication. The two-photon EPR-state was successfully employed for quantum teleportation, the transfer of an unknown quantum state to a remote location with perfect fidelity [19]. Multi-photon entangled states can be used for the generalization of the teleportation protocol. It is well known that the no cloning theorem prohibits perfect copying of unknown quantum states but it has been shown that this is universal quantum cloning machine (UQCM) which can reproduces two copies with an optimal fidelity of $5/6$ [6]. As it turns out that the quantum state (Eq. (7)) is needed to perform the so-called telecloning protocol, this protocol is defined as an optimal cloning of unknown quantum state at different locations [7].

In conclusion, we have shown that the parametric type-II down conversion not only produces entangled photon pairs, but also highly entangled four photon states well suited for new test of local realism and novel quantum communication schemes, in particular this state can be used for four-party secret sharing or for a three-party secret key distribution.

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