

Measurements of Nonlocal Variables and Demonstration of the Failure of the Product Rule for a Pre- and Postselected Pair of Photons

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We report the first implementation of the von Neumann instantaneous measurements of nonlocal variables, which becomes possible due to technological achievements in creating hyperentangled photons. Tests of reliability and of the nondemolition property of the measurements have been performed with high precision, showing the suitability of the scheme as a basic ingredient of numerous quantum information protocols. The method allows us to demonstrate for the first time with strong measurements a special feature of pre- and postselected quantum systems: the failure of the product rule. It has been verified experimentally that for a particular pre- and postselected pair of particles, a single measurement on particle A yields with certainty $\sigma_x^A = -1$, a single measurement on particle B yields with certainty $\sigma_y^B = -1$, and a single nonlocal measurement on particles A and B yields with certainty $\sigma_x^A \sigma_y^B = -1$.

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All known interactions in nature are local. It was thus believed (e.g., Ref. [1]) that measurements of nonlocal variables (variables that are related to more than one region of space) are impossible. However, Aharonov and his coauthors [2,3] showed theoretically that some nonlocal variables can be measured. For two separate locations, the sum of local variables, $A + B$, and the modular sum $(A + B) \bmod c$ are always measurable. On the other hand, they also showed that some other nonlocal variables cannot be measured as this would lead to superluminal signaling. Note that if we do not require the measurement to be nondemolition, then theoretically all nonlocal variables are measurable [4], but the procedure places high demands on entanglement resources [5–7].

Aharonov's main motivation was to shed light on relativistic quantum field theory [2,8–11], but the main impact of the analysis of measurements of nonlocal variables was in the field of quantum information [12–20]. In particular, it allowed an efficient method for teleportation [21] and was the basis for cryptographic protocols [22–25].

In this work, we demonstrate the measurement of nonlocal variables in its original sense, the one that is closest to the standard von Neumann definition of measurement in quantum mechanics [26]. Note that there exists an

alternative scheme [27] alongside a particular proposal for its implementation [28,29], which, however, has the drawback of being a probabilistic measurement; i.e., even with ideal devices it might not provide an outcome.

After performing and testing our measurement procedure, we apply it to show the peculiar phenomenon of the failure of the product rule for two separate (and thus commuting) local variables, which can take place only for pre- and postselected quantum systems [30–32]. There have been several demonstrations of the failure of the product rule for weak values—the outcomes of weak measurements [33–35] in the context of the Hardy paradox [30]. These, however, are very different results, obtained from many measurements on an ensemble of particles. In our scenario, we are able to violate the product rule using strong nondemolition measurements, providing direct information about single pairs of particles. We show below that the same cannot be done in the setting of the Hardy paradox.

Let us start by spelling out the properties of a von Neumann measurement. It has to be reliable, nondemolition, and instantaneous.

Reliability: If the initial state is an eigenstate of the measured nonlocal variable, the corresponding eigenvalue is found with certainty.

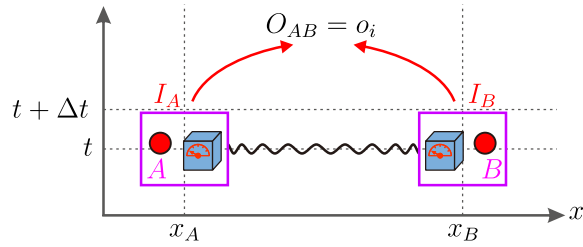


FIG. 1. Measurement scheme of a nonlocal variable. Entangled parts of the measuring device in locations A and B couple simultaneously to the respective parts of the system for a short time. The locally obtained records I_A and I_B provide, when brought together, the eigenvalue o_i of the nonlocal variable O_{AB} of the system.

Nondemolition property: If the initial state is an eigenstate of the measured nonlocal variable, the state is not changed. This ensures repeatability, namely, that identical consecutive measurements all agree with the reading of the first measurement.

Instantaneousness: The standard definition in nonrelativistic quantum mechanics is that the measurement process has a negligible duration. According to the principles of relativistic quantum mechanics, there is no physical mechanism that allows a local observer simultaneous coupling to spatially separated parts of a physical system. Thus, a measurement of a nonlocal observable must consist of two separate local couplings at distinct spatial locations followed by the creation of macroscopic records. In addition

to the requirement of negligible duration of these local couplings, together with the respective creation of local records, we require that the processes at the two locations are spacelike separated. The measurement coupling and creation of macroscopic records, which specify the result of the measurement, happen instantaneously and simultaneously in some Lorentz frame, e.g., that of the observer.

To satisfy the above requirements, our scheme for the measurement of nonlocal variables has two stages (see Fig. 1). In the first stage, spatially separated measuring devices (“pointers”) interact locally with the respective components of the system. In the second stage, immediately after, macroscopic records are created via local measurements of the measuring device. To gain nonlocal information from these local measurements, the crucial element in our scheme is the preparation of the measuring device in a nonlocal entangled state.

We perform a measurement of the product of polarization operators of two photons separated in space. The measuring device is given by the path degree of freedom (d.o.f.) of the photons themselves, which is measured interferometrically (see Fig. 2). In this setting, the local macroscopic records correspond to ports of the interferometer in which the detectors eventually click. The meaning of “nondemolition” is that the polarization state of the pair of photons in the corresponding pair of ports is still the initial (possibly entangled) polarization state as long as it starts in an eigenstate of the nonlocal variable. While in our experiment there is no spacelike separation of the local

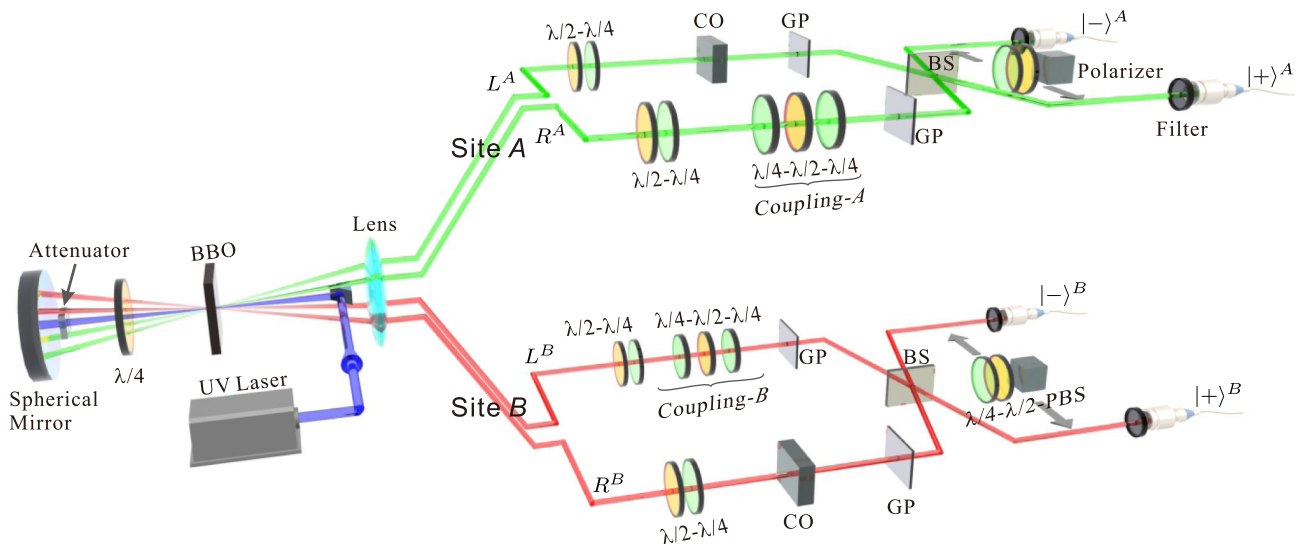


FIG. 2. Experimental setup. The two-photon, path-polarization, hyperentangled state is generated from the SPDC using β -BaB₂O₄ (BBO) crystal in a round-tripping confocal structure [36,37]. Photon A will be in the upper layer (green lines), while the lower layer is site B (red lines). The attenuator and sets of $\lambda/2$ - $\lambda/4$ wave plates prepare the polarization state. The local coupling between the system (polarization d.o.f.) and the measuring device (path d.o.f., L and R) is realized by sets of wave plates ($\lambda/4$ - $\lambda/2$ - $\lambda/4$) in a particular arm of the interferometer. For the nonlocal measurement, two such local couplings are applied. In the other arms, compensation crystals (CO) ensure identical optical path length. The relative phases are tuned using thin glass plates (GP) before bringing the arms to interference using a polarization insensitive beam splitter (BS). For testing the nondemolition property, polarization filters composed of a set of $\lambda/4$ - $\lambda/2$ plates and a polarization beam splitter (PBS) are placed in front of the detectors.

measurement processes, each one consisting of a coupling in one arm of the interferometer and the detector click, which provides the macroscopic record, our scheme in no way relies on the proximity of the two parties and thus allows for a modification with a spacelike separation of the local measurements.

The Hilbert spaces of photon polarization states and of the states of a spin $-\frac{1}{2}$ particle are isomorphic, which allows us to use a more familiar language of spin operators. Expressed with spin operators, our measurement corresponds to the measurement of $\sigma_z^A \sigma_z^B$ on two remote particles. There is no general method for the measurement of the product of variables belonging to remote locations; however, $\sigma_z^A \sigma_z^B = (\sigma_z^A + \sigma_z^B) \bmod 4 - 1$, so the measurement of this product is equivalent to the measurement of a modular sum, which is proven to be possible [3]. For the polarization d.o.f., we use the correspondence

$$\sigma_z |\uparrow_z\rangle = |\uparrow_z\rangle \leftrightarrow |H\rangle, \quad |\downarrow_z\rangle \leftrightarrow |V\rangle, \quad (1)$$

with $|H\rangle$ and $|V\rangle$ identifying horizontal and vertical polarization, respectively. Similarly, for the measuring device, which we mark with “ \sim ” in the spin notation,

$$\tilde{\sigma}_z |\tilde{\uparrow}_z\rangle = |\tilde{\uparrow}_z\rangle \leftrightarrow |L\rangle, \quad |\tilde{\downarrow}_z\rangle \leftrightarrow |R\rangle, \quad (2)$$

where $|L\rangle$ and $|R\rangle$ denote the two arms the photons can take at both locations.

The measurement scheme requires us to correctly match the preparation of the initial pointer state, the interaction Hamiltonian, and the pointer measurement. We prepare the measuring device in the entangled state,

$$|\Psi^+\rangle^{AB} \equiv \frac{1}{\sqrt{2}} (|\tilde{\uparrow}_z\rangle^A |\tilde{\downarrow}_z\rangle^B + |\tilde{\downarrow}_z\rangle^A |\tilde{\uparrow}_z\rangle^B), \quad (3)$$

and perform local pointer measurements in the plane perpendicular to $\tilde{\sigma}_z$. The interaction Hamiltonian is

$$H = g(t) [(\mathbb{1}^A - \sigma_z^A)(\mathbb{1}^B - \tilde{\sigma}_z^B) + (\mathbb{1}^A + \sigma_z^A)(\mathbb{1}^B + \tilde{\sigma}_z^B)], \quad (4)$$

where $\int g(t) dt = \pi/4$.

Given a polarization state that is an eigenstate of $\sigma_z^A \sigma_z^B$ with eigenvalue $+1$, the interaction Hamiltonian (4) does not change the state $|\Psi^+\rangle^{AB}$ of the measuring device. On the other hand, for an eigenstate with eigenvalue -1 , the state of the measuring device is changed to

$$|\Psi^-\rangle^{AB} \equiv \frac{1}{\sqrt{2}} (|\tilde{\uparrow}_z\rangle^A |\tilde{\downarrow}_z\rangle^B - |\tilde{\downarrow}_z\rangle^A |\tilde{\uparrow}_z\rangle^B). \quad (5)$$

The states $|\Psi^+\rangle^{AB}$ and $|\Psi^-\rangle^{AB}$ can be distinguished reliably using our simultaneous local measurements in A and in B , after the records have been combined. In both cases, after the interaction, the system and the pointer are not entangled

with each other and the system state is not changed. This makes the measurement nondemolition. Note that the product of spin operators in other directions can also be measured in this way, when the Hamiltonian is adapted accordingly. In addition, a strictly local spin measurement can be achieved using this scheme when only one of the two local coupling terms is present in the interaction Hamiltonian.

In our experiment (as shown in Fig. 2), a vertically polarized ultraviolet ($\lambda = 406.7$ nm) laser beam is focused and reflected to pump a 0.5-mm-thick BBO crystal (type-1 cut at 29.11°). Because of the degenerate spontaneous parametric down-conversion (SPDC), horizontally polarized photon pairs are emitted in a cone with apex angle 3° (postselected by spectral filters centered at $\lambda = 813.4$ nm with bandwidth $\Delta\lambda = 3$ nm). After passing a wave plate, acting as $\lambda/4$ for 813.4 nm light, with optical axis oriented at 45° , both the pump laser and the emitted photons are reflected by a spherical mirror with a 150-mm radius. While the polarization of previously produced photon pairs is converted to vertical, a second pass through the BBO of the pump can produce another pair of horizontally polarized photons.

The two processes overlap both spatially and temporally due to the confocal structure and the long coherence time. A positive lens ($f = 150$ mm) transforms the conical parametric emission into a cylindrical one, preparing a so-called entanglement ring [37]. By selecting four points in the ring, we obtain the two-photon four-qubit hyperentangled state, maximally entangled both in the polarization and spatial d.o.f., while separable between the two. The spatial state reads $|\Psi^+\rangle^{AB} = (1/\sqrt{2})(|L\rangle^A |R\rangle^B + |R\rangle^A |L\rangle^B)$. An attenuator near the spherical mirror together with the wave-plate sets in the four arms of the interferometer allow us to prepare arbitrary polarization states.

The two parts of the interaction Hamiltonian of Eq. (4) at the two sites A and B are each implemented by sets of wave plates, introducing a conditional phase shift of π for horizontally polarized photons in the right arm at site A and in the left arm at site B . Using glass plates, we tune the interferometer such that without interaction we get complete correlation between clicks of the detectors in sites A and B . Then, observing the clicks corresponds to a “spin” measurement in some unknown direction in the $x-y$ plane, which, however, is the same for the two particles. In the notation of the photon states, these are the measurements in the basis $|\pm\rangle \equiv (1/\sqrt{2})(|L\rangle \pm e^{i\varphi}|R\rangle)$ with unknown phase φ , which is equal at both sites. In this notation, the states of the measuring device (3) and (5) are

$$|\Psi^+\rangle^{AB} = \frac{e^{-2i\varphi}}{\sqrt{2}} (|+\rangle^A |+\rangle^B - |-\rangle^A |-\rangle^B), \quad (6a)$$

$$|\Psi^-\rangle^{AB} = \frac{e^{-2i\varphi}}{\sqrt{2}} (|+\rangle^A |-\rangle^B - |-\rangle^A |+\rangle^B), \quad (6b)$$

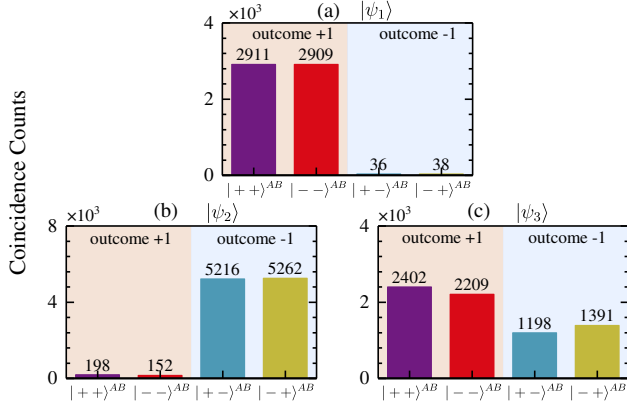


FIG. 3. Reliability test. Coincidence counts on various pairs of detectors for the three polarization states of Eq. (7). The states $|\psi_1\rangle$ and $|\psi_2\rangle$ are eigenstates of the nonlocal variable $\sigma_z^A \sigma_z^B$. The percentage of outcomes for state $|\psi_3\rangle$ corresponds well to the initial superposition.

where we used the shortcut notation $|+\rangle^{AB} \equiv |+\rangle^A |+\rangle^B$, etc. Thus, the correlations and the anticorrelations of the outcomes for the measurements of $|+\rangle$ and $|-\rangle$ in the path d.o.f. allow us to distinguish between the eigenstates $+1$ and -1 of the product operator of the system.

We test the reliability and the nondemolition property of our measurement procedure for three different states,

$$|\psi_1\rangle = |\uparrow_z\rangle^A |\uparrow_z\rangle^B, \quad (7a)$$

$$|\psi_2\rangle = \frac{1}{\sqrt{2}} (|\uparrow_z\rangle^A |\downarrow_z\rangle^B + i |\downarrow_z\rangle^A |\uparrow_z\rangle^B), \quad (7b)$$

$$|\psi_3\rangle = \sqrt{0.5} |\uparrow_z\rangle^A |\uparrow_z\rangle^B + \sqrt{0.1} |\uparrow_z\rangle^A |\downarrow_z\rangle^B - i \sqrt{0.2} |\downarrow_z\rangle^A |\uparrow_z\rangle^B + \sqrt{0.2} |\downarrow_z\rangle^A |\downarrow_z\rangle^B. \quad (7c)$$

The results of the reliability test are presented in Fig. 3. For $|\psi_1\rangle$ and $|\psi_2\rangle$, only 1.3% and 3.2% of the events are erroneous, respectively. State $|\psi_3\rangle$, which is not an eigenstate of the nonlocal operator, leads to 64% events corresponding to outcome $+1$ and 36% to outcome -1 , roughly agreeing with the theoretical predictions of 70% and 30%, respectively.

To confirm that the measurement is nondemolition, we perform state verification measurements by adding polarization analyzers directly in front of the detectors. We test the polarization in one of the pairs of output ports, between which coincidences are expected, and repeat the test for the same measurement time with differently set filters. For the state $|\psi_1\rangle$, the results are very robust, as shown in Fig. 4(a), as only 0.2% changed their polarization. This measurement verifies that the output state is indeed $|\uparrow_z\rangle^A |\uparrow_z\rangle^B$. The entangled state $|\psi_2\rangle$ requires two tests. The first measurement in the basis σ_z^A, σ_z^B shows that the state corresponding to the -1 outcome ($|+\rangle^{AB}$) of $|\psi_2\rangle$ is in a superposition

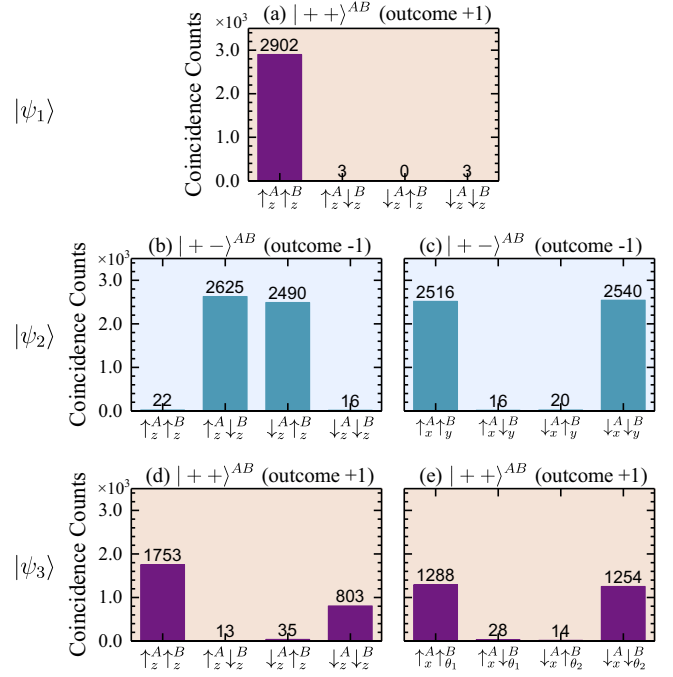


FIG. 4. Test of nondemolition property. (a) $|\psi_1\rangle$ is measured on the detectors $|+\rangle^{AB}$ in the σ_z^A, σ_z^B basis using polarization filters to verify that the state is unaltered during the coupling. (b,c) For $|\psi_2\rangle$, the measurements were done on detectors $|+\rangle^{AB}$ in bases σ_z^A, σ_z^B and σ_x^A, σ_y^B . (d) State $|\psi_3\rangle$ was measured first in the σ_z^A, σ_z^B basis. To fix the phase relation between $|\uparrow_z\rangle^A |\uparrow_z\rangle^B$ and $|\downarrow_z\rangle^A |\downarrow_z\rangle^B$, a second measurement (e) was conducted with projections also containing rotated states with $|\uparrow_{\theta_1}\rangle = (\sqrt{0.5} |\uparrow_z\rangle + \sqrt{0.2} |\downarrow_z\rangle) / \sqrt{0.7}$ and $|\uparrow_{\theta_2}\rangle = (\sqrt{0.5} |\uparrow_z\rangle - \sqrt{0.2} |\downarrow_z\rangle) / \sqrt{0.7}$. The results prove that we performed nondemolition measurements of nonlocal variables.

(or mixture) of $|\uparrow_z\rangle^A |\downarrow_z\rangle^B$ and $|\downarrow_z\rangle^A |\uparrow_z\rangle^B$ according to Fig. 4(b). The second measurement in basis σ_x^A, σ_y^B (with $\sigma_x \equiv |\downarrow_z\rangle \langle \uparrow_z| + |\uparrow_z\rangle \langle \downarrow_z|$ and $\sigma_y \equiv i |\downarrow_z\rangle \langle \uparrow_z| - i |\uparrow_z\rangle \langle \downarrow_z|$) [see Fig. 4(c)] fixes the phase relation between those contributions. In both bases, we get only 0.7% errors. For state $|\psi_3\rangle$, we test the projection onto the subspace corresponding to outcome $+1$ in the ports $|+\rangle^{AB}$. Again, we observe a good correspondence with theoretical predictions [see Figs. 4(d) and 4(e)].

In the second part of the experiment, we use our technique of measurement of nonlocal variables to demonstrate the failure of a product rule for pre- and post-selected quantum systems.

Before presenting our measurement, we show that the failure of the product rule appearing in the Hardy paradox [30] (which was demonstrated using weak measurements [33–35]) cannot be observed using von Neumann measurement. Such a measurement would allow superluminal communication between Alice at site A and Bob at remote site B . In this case, Alice would be able to change the probability distribution at Bob's site instantaneously.

Assume that a nondemolition measurement of $\mathbf{P}_1^A \mathbf{P}_1^B$ is performed at time t with $\mathbf{P}_1^A \equiv |1\rangle\langle 1|^A$ and $\mathbf{P}_1^B \equiv |1\rangle\langle 1|^B$. Just before t , Bob prepares the state $(1/\sqrt{2})(|0\rangle^B + |1\rangle^B)$ and performs a projection measurement of this state immediately after time t . If, also just before t , Alice prepares state $|0\rangle^A$, then at time t the state in the two sites is an eigenstate of $\mathbf{P}_1^A \mathbf{P}_1^B$ with eigenvalue 0, so the nonlocal measurement will not change the state and Bob will find $(1/\sqrt{2})(|0\rangle^B + |1\rangle^B)$ with probability 1. If, instead, Alice, just before t , prepares state $|1\rangle^A$, the nonlocal measurement will either lead to state $|1\rangle^A|0\rangle^B$ for the outcome $\mathbf{P}_1^A \mathbf{P}_1^B = 0$ or to state $|1\rangle^A|1\rangle^B$ for the outcome $\mathbf{P}_1^A \mathbf{P}_1^B = 1$. In both cases, the probability for Bob to find the state $(1/\sqrt{2})(|0\rangle^B + |1\rangle^B)$ is $\frac{1}{2}$, and Alice would have changed the probability distribution of Bob's outcomes instantaneously by her state preparation.

For our demonstration of the failure of the product rule, we measure the product $\sigma_x^A \sigma_y^B$, instead of measuring a product of projection operators, which *is* measurable in the von Neumann sense. In our example, we have a pair of particles for which we know with certainty that a single local measurement of σ_x^A or of σ_y^B results in the outcome -1 . If, however, we measure the product $\sigma_x^A \sigma_y^B$, it will also provide the outcome -1 with certainty, instead of the naively expected product outcome $+1$.

To this end, we preselect the singlet state $(1/\sqrt{2})(|\uparrow_z\rangle^A |\downarrow_z\rangle^B - |\downarrow_z\rangle^A |\uparrow_z\rangle^B)$ and postselect the product state $|\uparrow_y\rangle^A |\uparrow_x\rangle^B$ [31]. Indeed, the initial state is anticorrelated for all directions, so $\sigma_x^A = -1$ follows immediately from postselection of $|\uparrow_x\rangle^B$ and $\sigma_y^B = -1$ follows immediately from postselection of $|\uparrow_y\rangle^A$. We can see that $\sigma_x^A \sigma_y^B = -1$ with certainty by noticing that the outcome $+1$ can never occur. If prior to the postselection the nonlocal measurement yields $\sigma_x^A \sigma_y^B = +1$, the initial state is projected on the space spanned by states $|\uparrow_x\rangle^A |\uparrow_y\rangle^B$ and $|\downarrow_x\rangle^A |\downarrow_y\rangle^B$. Then, in the basis of σ_y^A and σ_x^B , the projected state is $-(1+i)/2(|\uparrow_y\rangle^A |\downarrow_x\rangle^B + |\downarrow_y\rangle^A |\uparrow_x\rangle^B)$. Since it is orthogonal to the postselected state $|\uparrow_y\rangle^A |\uparrow_x\rangle^B$, the postselection will never be successful, and thus the outcome of the measurement will always be $\sigma_x^A \sigma_y^B = -1$.

To implement this measurement of $\sigma_x^A \sigma_y^B$, we only need a simple modification of our measurement scheme. The interaction Hamiltonian is changed to

$$H = g(t)[(\mathbb{1}^A - \sigma_x^A)(\mathbb{1}^A - \tilde{\sigma}_z^A) + (\mathbb{1}^B - \sigma_y^B)(\mathbb{1}^B + \tilde{\sigma}_z^B)], \quad (8)$$

which is achieved experimentally in a straightforward way by modifying the settings of the wave plates at the two sites accordingly. The measurement procedure is then completely analogous to the measurement of $\sigma_z^A \sigma_z^B$. For the local measurements of σ_x^A and σ_y^B , the coupling at the

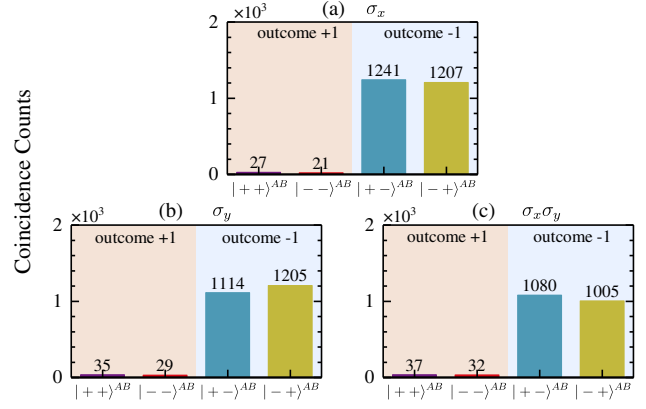


FIG. 5. Experimental results for demonstrating the failure of the product rule. Coincidence counts in pairs of detectors for measurement of local and nonlocal variables: (a) σ_x^A , (b) σ_y^B , (c) $\sigma_x^A \sigma_y^B$. In all three cases, the system is preselected in a singlet state and postselected in the product state $|\uparrow_y^A \downarrow_x^B\rangle$.

respective opposite site is switched off by setting the optical axis of the wave plates at zero, such that the interaction Hamiltonians become $H = g(t)(\mathbb{1}^A - \sigma_x^A)(\mathbb{1}^A - \tilde{\sigma}_z^A)$ and $H = g(t)(\mathbb{1}^A - \tilde{\sigma}_z^A)(\mathbb{1}^B + \tilde{\sigma}_z^B)$.

The results are presented in Fig. 5. As in the nonlocal measurement of $\sigma_z^A \sigma_z^B$ explained above, the correlations and the anticorrelations of the outcomes for the measurements of $|+\rangle$ and $|-\rangle$ in the path d.o.f. allow us to distinguish between the eigenstates $+1$ and -1 of the product operator of the system. Figure 5(a) shows $\sigma_x^A \sigma_y^B = -1$ with very high precision. For the local measurement at site A (B), we use the same procedure without the interaction at site B (A) such that, again, the observed anticorrelations, Figs. 5(b) and 5(c), clearly demonstrate $\sigma_x^A = -1$ and $\sigma_y^B = -1$.

We have performed the first experimental implementation of von Neumann measurements of nonlocal variables. We also demonstrated a peculiar property of pre- and postselected composite systems: the failure of the product rule for commuting observables. The experiment sheds light on the fundamental question of the physical meaning of nonlocal variables. The high fidelity of the experimental results demonstrates that the method is potentially useful for quantum information applications.

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