Multipartite Entanglement Detection with Minimal Effort

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Certifying entanglement of a multipartite state is generally considered a demanding task. Since an N qubit state is parametrized by $4^N - 1$ real numbers, one might naively expect that the measurement effort of generic entanglement detection also scales exponentially with N. Here, we introduce a general scheme to construct efficient witnesses requiring a constant number of measurements independent of the number of qubits for states like, e.g., Greenberger-Horne-Zeilinger states, cluster states, and Dicke states. For four qubits, we apply this novel method to experimental realizations of the aforementioned states and prove genuine four-partite entanglement with two measurement settings only.

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Introduction.—Entanglement is a fascinating feature of strictly quantum nature. It was first studied for the bipartite case [1,2] and has already been applied for first quantum communication tasks like quantum cryptography and quantum teleportation [3]. The generalization to multipartite entanglement comes with a whole new set of features providing, relative to separable states, information processing advantages for quantum computation and simulation or for quantum metrology. It is thus crucial to have tools at hand which allow us to identify *genuinely multipartite entangled* states [4–6].

Proving genuine multiparty entanglement is in general a complex task. Full quantum state tomography (QST) can be used for detecting and even for quantifying entanglement, but requires the determination of exponentially many parameters. Even when using simplified procedures [7–9], the effort is still significant. Thus, the goal was to find a direct measurement procedure for witnessing entanglement [5,10–13]. The only systematic method known today for constructing entanglement witnesses uses the fidelity relative to a chosen reference state. However, depending on the state, this as well leads to a rapidly increasing number of measurements required to infer the fidelity. Remarkably, specifically for the cluster and Greenberger-Horne-Zeilinger (GHZ) states, witnesses based on the stabilizer formalism [14] have been found incidentally which require only two measurements for any number of qubits [15]. Still, a systematic method, also not restricted to stabilizer states, is missing.

In this Letter, we introduce a constructive scheme to derive efficient multipartite entanglement witnesses, i.e., witnesses which can be evaluated from only a very small number of measurements. Our scheme employs basic properties of operators and their expectation values to construct witnesses for many relevant quantum states which require only two measurement settings, *independent* of the number of qubits. We show a way to enhance the concept of finding measurements that are complementary for separable states [16] by introducing weights and providing the alternative scheme of testing violation of a set of inequalities in order to further increase the sensitivity. We demonstrate how to derive these efficient entanglement criteria for several of the most prominent quantum states, encompassing GHZ and cluster states, Dicke and W states, and the multipartite singlet state.

Every quantum mechanical N-qubit state ρ is uniquely described by its correlation tensor T,

$$\rho = \frac{1}{2^N} \sum_{j \in \mathcal{I}} T_j \sigma_j, \tag{1}$$

where the set $\mathcal{I} = \{0...00, 0...01, ..., 3...33\}$ labels all indices $j = (j_1 ... j_N)$, $j_i \in \{0, 1, 2, 3\}$ of the correlation tensor with $\sigma_j = \sigma_{j_1} \otimes ... \otimes \sigma_{j_N}$ and with Pauli matrices $\sigma_0, \sigma_1, \sigma_2$, and σ_3. The correlation tensor elements (for short called *correlations*) are given by $T_j = \langle \sigma_j \rangle = \mathrm{Tr}[\rho \sigma_j]$. Since the eigenvalues of σ_j are ± 1, the correlations are constrained to lie in the interval $[-1, 1]$ and consequently $T_j^2 \leq 1$. These constraints, together with the physicality condition $\rho \geq 0$ imply various bounds on the summed squares of correlations, which are helpful for the construction of efficient witness operators. Consider a set of n pairwise commuting operators $\{\sigma_j : j \in \mathcal{C} \subset \mathcal{I}\}$. These operators have common eigenstates, for which $T_j = \pm 1$ holds. Consequently, the sum of squared correlations is bounded by $\sum_{j \in \mathcal{C}} T_j^2 \leq n$. On the contrary, for a set of pairwise anticommuting operators, e.g., $\{\sigma_j : j \in \mathcal{A} \subset \mathcal{I}\}$, the threshold is [16]

$$\sum_{j \in \mathcal{A}} T_j^2 \leq 1, \tag{2}$$

establishing a complementarity relation between the correlations [17].

Separability.—Consider the bipartition (*cut*) $\mathcal{B} = A|B$ of a multipartite quantum system into parts A and B.

Two operators given by $\sigma_{ab} = \sigma_a \otimes \sigma_b$ and $\sigma_{a'b'} = \sigma_{a'} \otimes \sigma_{b'}$ *anticommute with respect to the bipartition* $\mathcal{B}$ if $\{\sigma_a, \sigma_{a'}\} = 0$ or $\{\sigma_b, \sigma_{b'}\} = 0$, i.e., if they locally anticommute on A or on B. According to Ref. [16] this property is called *cut-anticommutativity* or, more specifically, $A|B$-anticommutativity. Since for states separable with respect to $\mathcal{B}$ the correlation tensor factorizes, $T_{ab} = T_a T_b$, these states fulfill

$$T_{ab}^2 + T_{a'b'}^2 \leq_{\underset{\mathcal{B}}{\mathrm{SEP}}} 1. \tag{3}$$

However, cut-anticommuting operators can also commute, i.e., $[\sigma_{ab}, \sigma_{a'b'}] = 0$, allowing the common (entangled) eigenstates of σ_{ab} and $\sigma_{a'b'}$ to exhibit $T_{ab}^2 + T_{a'b'}^2 > 1$. Therefore, violation of Eq. (3) rules out separability with respect to cut $\mathcal{B}$.

Testing entanglement.—To prove genuine multipartite entanglement of a state, Eq. (3) has to be violated for every possible bipartition. One starts with a list $\{\sigma_j\}$ of all operators with nonvanishing expectation value, $T_j \neq 0$ (all nonvanishing correlations). For the construction of the efficient entanglement criterion for a bipartition $\mathcal{B}$, one then chooses from that list two operators which are mutually commuting, but also cut-anticommuting relative to the bipartition $A|B$. One repeats this, until all bipartitions are tested.

The scheme becomes highly efficient if the correlation values of several σ_j can be obtained from the same measurement setting. In detail, this means that one makes use of the observation that from a single measurement setting $\mathcal{M}_k$ with $k = (k_1, k_2, \ldots, k_N)$ and $k_i \in \{1, 2, 3\}$ labeling the local Pauli measurements, all 2^N correlations T_j with $j \in \{(0, 0, \ldots, 0), (0, 0, \ldots, k_N), \ldots, (k_1, k_2, \ldots, k_N)\}$ can be inferred. Depending on the symmetry of the state, two measurement settings can suffice to prove genuine multipartite entanglement if one finds for each bipartition operators in the set that are commuting, but cut-anticommuting for the given bipartition.

Combined entanglement witness.—Combining the above criteria into a single witness facilitates the practical application (only a single value has to be calculated), though at the expense of a lower sensitivity, i.e., a reduced robustness against (white) noise. Compared to Ref. [16], the sensitivity can be considerably improved by using a weighted sum,

$$\mathcal{W} = \frac{1}{G_0} \sum_{j \in \mathcal{S}} v_j T_j^2 \leq_{\mathrm{BISEP}} \frac{G}{G_0}, \tag{4}$$

where $\mathcal{S} \subset \mathcal{I}$ labels the set of correlations that can be determined by the given set of measurements and where $\leq_{\mathrm{BISEP}}$ denotes that the inequality is valid for all biseparable states. The weights v_j and the (normalization) constants G (G_0) are determined as follows:

(i) Depict the operators defined by $\mathcal{S}$ as vertices of a graph (*anticommutativity graph*).

(ii) Assign weights $v_j > 0$ to the vertices.

(iii) Choose bipartition $\mathcal{B}_r$ and connect all vertices for which the corresponding operators cut-anticommute by edges. (If all operators indexed by $\mathcal{S}$ mutually commute, no edges will occur.) Distribute values $c_j^{(m)} = \{0, 1\}$ among vertices under the constraint that any two "1's" are not connected by an edge and calculate for each of the m possible distributions of 1's the sum $G_r^{(m)} = \sum_{j \in \mathcal{S}} c_j^{(m)} v_j$. The case of no partition will be labeled by $r = 0$. Repeat step (iii) for all bipartitions $\mathcal{B}_r$.

(iv) Every choice of weights v_j in Eq. (4) defines a witness with $G = \max_{r>0,m} G_r^{(m)}$ and $G_0 = \max_m G_0^{(m)}$. The ratio G/G_0 determines the noise robustness of the criterion. To optimize the witness in terms of its noise robustness, one has to choose the weights v_j according to $\arg\min_{\{v_j\}} G/G_0$.

Example.—Let us consider the four-party GHZ state $1/\sqrt{2}(|0000\rangle + |1111\rangle)$, whose nonvanishing correlations are listed in Table I. As one can see, the measurement of the single setting $\mathcal{M}_{3333}$ provides seven correlations with squared value 1. Since the operators of these correlations exhibit the same cut-anticommutation relation with any operator corresponding to the other eight correlations of Table I, the second measurement can be chosen arbitrarily out of those remaining eight. For example, the choice $\mathcal{M}_{1221}$ for the second measurement setting results in the set of operators $\{\sigma_{3333}, \sigma_{3300}, \sigma_{0033}, \sigma_{3003}, \sigma_{0330}, \sigma_{3030}, \sigma_{0303}, \sigma_{1221}\}$, i.e., $\mathcal{S} = \{3333, 3300, \ldots, 1221\}$.

States that are, e.g., $A|BCD$-separable fulfill, according to Eq. (3),

$$T_{3333}^2 + T_{1221}^2 \leq_{\underset{A|BCD}{\mathrm{SEP}}} 1. \tag{5}$$

Since σ_{1221} not only $A|BCD$ anticommutes with σ_{3333}, but also with $\sigma_{3030}, \sigma_{3003}, \sigma_{3300}$ from our list, a natural choice is to average over the expectation values of those four possibilities. Nonseparability against the partition $A|BCD$ can then be detected with

$$\mathcal{W}_{A|BCD}^{\mathrm{GHZ}} = \frac{1}{2}\left[\frac{1}{4}(T_{3030}^2 + T_{3003}^2 + T_{3300}^2 + T_{3333}^2) + T_{1221}^2\right] \leq_{\underset{A|BCD}{\mathrm{SEP}}} \frac{1}{2}, \tag{6}$$

where the additional normalization constant of $1/2$ is introduced to ensure that $\mathcal{W}_{A|BCD}^{\mathrm{GHZ}} = 1$ holds for the ideal GHZ state, where all squared expectation values are one. The criteria for the remaining six bipartitions are derived analogously. For the list of criteria for the four-qubit Dicke, singlet, and W state see the Supplemental Material [18].

To derive a combined entanglement witness for the GHZ state, we use all eight operators labeled by $\mathcal{S}$ (see Table I). We assign equal weights to the seven operators obtained

TABLE I. All nonvanishing correlations of the four-qubit GHZ state. The correlations in the first two rows can be infered from the measurement setting $\mathcal{M}_{3333}$ and the last correlation is obtained from the setting $\mathcal{M}_{1221}$.

T_{0000}	1	$\boldsymbol{T}_{0033}$	1	$\boldsymbol{T}_{0303}$	1	$\boldsymbol{T}_{0330}$	1
$\boldsymbol{T}_{3003}$	1	$\boldsymbol{T}_{3030}$	1	$\boldsymbol{T}_{3300}$	1	$\boldsymbol{T}_{3333}$	1
T_{2112}	−1	T_{2121}	−1	T_{2211}	−1	T_{2222}	1
T_{1111}	1	T_{1122}	−1	T_{1212}	−1	$\boldsymbol{T}_{1221}$	−1

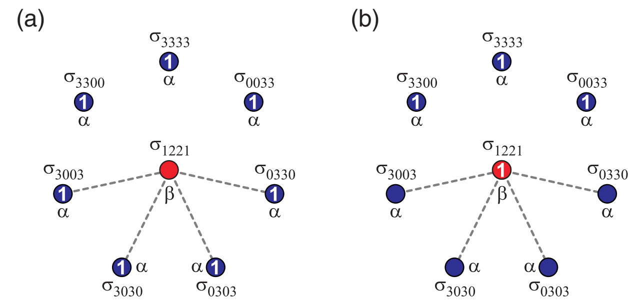

FIG. 1. The operators used to construct the witness $\mathcal{W}^{\mathrm{GHZ}}$, cf. Table I. As an example, the cut-anticommutation relations for the cut $AB|CD$ are indicated by dashed lines. One realizes that for each bipartition four of the seven operators obtained from the measurement of setting $\mathcal{M}_{3333}$ cut-anticommute with σ_{1221}. Thus, the same weights α are assigned to them, while σ_{1221} is weighted with β. Depending on the distribution of 1's, the sum for this bipartition is found to be either (a) $\tilde{G}_r^{(1)}=7\alpha$ or (b) $\tilde{G}_r^{(2)}=3\alpha+\beta$. The best weights are obtained when the two assignments are equally good, i.e., $7\alpha=3\alpha+\beta$.

from the measurement setting $\mathcal{M}_{3333}$, i.e., $\alpha=v_{3333}=v_{0033}=\cdots=v_{3300}$ since these mutually commute and behave similarly with regard to the cut-anticommutation relations with σ_{1221} for the different bipartitions. The weight of the remaining operator will be denoted by $\beta=v_{1221}$. From the anticommutativity graph (one without any edges) one obtains $G_0=7\alpha+\beta$. Depending on the distribution of 1's, the sums for all bipartitions are either $G_r^{(1)}=7\alpha$ or $G_r^{(2)}=3\alpha+\beta$, see Fig. 1. For optimal noise robustness, one has to find the weights v_j by minimizing G/G_0. The minimum is achieved for $G_r^{(1)}=G_r^{(2)}$, thus $7\alpha=3\alpha+\beta$, which leads, by arbitrarily setting $\alpha=1$, to $G_0=7\alpha+\beta=7+4=11$ and $G=7\alpha=3\alpha+\beta=7$. Then, the optimized two-measurement witness for the GHZ state reads

$$\mathcal{W}^{\mathrm{GHZ}}=\frac{1}{11}(T_{3333}^2+T_{3300}^2+T_{0033}^2+T_{3003}^2+T_{0330}^2+T_{3030}^2+T_{0303}^2+4T_{1221}^2)\leq_{\mathrm{BISEP}}\frac{7}{11}. \quad (7)$$

Analogously, for the cluster state $|\mathcal{C}_4\rangle\propto(|0000\rangle+|0011\rangle-|1100\rangle+|1111\rangle)$ one obtains the witness

$$\mathcal{W}^{\mathcal{C}_4}=\frac{1}{6}(T_{3300}^2+T_{3011}^2+T_{0311}^2+T_{1130}^2+T_{1103}^2+T_{0033}^2)\leq_{\mathrm{BISEP}}\frac{2}{3}. \quad (8)$$

For details on the derivation, see the Supplemental Material [18].

Extensions.—Similar criteria can also be formulated for more qubits. The two-measurement-witness for the N-qubit GHZ state is based upon the measurements of $\mathcal{M}_{3333\ldots3}$ and, e.g., $\mathcal{M}_{2211\ldots1}$ since one is able to find operators whose expectation value can be determined by those measurements such that Eq. (3) can be violated for each bipartition. Then, genuine multipartite entanglement is detected by violation of

$$\mathcal{W}^{\mathrm{GHZ}_N}=\frac{1}{2^{N-1}+2^{N-2}-1}[T_{3333\ldots3}^2+T_{0033\ldots3}^2+T_{0303\ldots3}^2+\cdots+T_{33\ldots300}^2+2^{N-2}T_{2211\ldots1}^2]\leq_{\mathrm{BISEP}}\frac{2^{N-1}-1}{2^{N-1}+2^{N-2}-1}\underset{N\to\infty}{\rightarrow}\frac{2}{3}. \quad (9)$$

The extension of the criterion for the N qubit cluster state $|\tilde{\mathcal{C}}_N\rangle$ (N even) is based on the correlations $\{T_j|j\in\mathcal{S}_{1313\ldots13}\cup\mathcal{S}_{3131\ldots31}\}$ where the set $\mathcal{S}_k$ indexes all non-vanishing correlations of the cluster state that can be determined from the measurement setting $\mathcal{M}_k$. Please note that $|\tilde{\mathcal{C}}_4\rangle$ as defined via the stabilizer formalism [14] equals $|\mathcal{C}_4\rangle$ up to LU transformations. Genuine multipartite entanglement of $|\tilde{\mathcal{C}}_N\rangle$ is then identified by violation of

$$\mathcal{W}^{\tilde{\mathcal{C}}_N}=\frac{\sum_{j\in\mathcal{S}_{1313\ldots13}}T_j^2+\sum_{j\in\mathcal{S}_{3131\ldots31}}T_j^2}{2(2^{N/2}-1)}\leq_{\mathrm{BISEP}}\frac{2^{N/2-1}+2^{N/2}-2}{2(2^{N/2}-1)}\underset{N\to\infty}{\rightarrow}\frac{3}{4}. \quad (10)$$

Analysis of experimental data.—In order to experimentally demonstrate the applicability of our new entanglement criteria, we prepare a series of superpositions of GHZ and

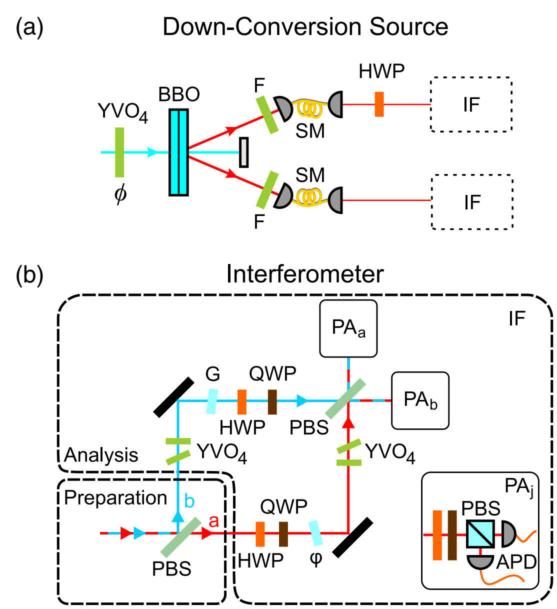

FIG. 2. Scheme of the experimental setup. In a first step (a) a type-I SPDC source together with a half wave plate (HWP) at angle θ is used to prepare states of the form $(|H\rangle(\cos2\theta|H\rangle+\sin2\theta|V\rangle)+e^{i\phi}|V\rangle(-\cos2\theta|V\rangle+\sin2\theta|H\rangle))/\sqrt{2}$. The phase ϕ can be set by a birefringent yttrium-vanadate crystal (YVO_4). Interference filters (F) are applied for spectral filtering and spatial filtering is performed by coupling into single mode fibers (Supplemental Material [18]). In a second step (b), the state preparation is completed by increasing the Hilbert space by polarizing beam splitters (PBSs). Overlap at a beam splitter and polarization analysis allows us to measure all Pauli settings σ_i and to perform QST. YVO_4 crystals and glass plates (G and φ) inside the interferometer are used for phase and path length compensation, respectively.

TABLE II. Experimental values of the individual criteria and combined witnesses for the considered states. All values for a specific bipartition are clearly above the threshold of $1/2$, indicating genuine four-partite entanglement in all cases. The thresholds for the combined criteria are $7/11$ (GHZ), $2/3$ (cluster), $4/5$ ($D_4^{(2)}$), and $3/5$ (Ψ_4) respectively. The Dicke state cannot be significantly proven to be genuinely four-partite entangled by means of the combined witness, see Supplemental Material [18]. Hence, one has to resort to the individual criteria in this case.

Partition	$\lvert\mathrm{GHZ}\rangle$	$\lvert\mathcal{C}_4\rangle$	$\lvert D_4^{(2)}\rangle$	$\lvert\Psi_4\rangle$
$A\vert BCD$	0.894 ± 0.007	0.922 ± 0.006	0.819 ± 0.013	0.804 ± 0.019
$B\vert ACD$	0.906 ± 0.006	0.940 ± 0.004	0.819 ± 0.013	0.804 ± 0.019
$C\vert ABD$	0.906 ± 0.006	0.940 ± 0.004	0.819 ± 0.013	0.804 ± 0.019
$D\vert ABC$	0.906 ± 0.006	0.928 ± 0.006	0.819 ± 0.013	0.804 ± 0.019
$AB\vert CD$	0.904 ± 0.006	0.922 ± 0.006	0.627 ± 0.013	0.608 ± 0.017
$AC\vert BD$	0.906 ± 0.006	0.948 ± 0.004	0.620 ± 0.013	0.594 ± 0.021
$AD\vert BC$	0.901 ± 0.006	0.943 ± 0.004	0.625 ± 0.013	0.622 ± 0.021
Combined	0.916 ± 0.005	0.940 ± 0.004	0.801 ± 0.017	0.683 ± 0.014

cluster states with variable weights. Different linear optical setups to prepare either four-qubit GHZ [21] or cluster states [22] are known. To have the flexibility to prepare superpositions of GHZ and cluster states in a single setup, we resort to a two photon experiment using two degrees of freedom per photon, namely polarization and path [23]. This approach enables one to prepare states with both high fidelity and high count rates. From now on, the computational basis states $|0\rangle$ and $|1\rangle$ are encoded either in polarization or in the path degree of freedom, i.e., $|0\rangle \to |H\rangle$ and $|1\rangle \to |V\rangle$ for horizontal (H) and vertical (V) polarization and $|0\rangle \to |a\rangle$ and $|1\rangle \to |b\rangle$ for paths a and b.

The photon source shown in Fig. 2(a) uses spontaneous parametric down-conversion and allows us to prepare states of the form $(|H\rangle(\cos(2\theta)|H\rangle + \sin(2\theta)|V\rangle) + e^{i\phi}|V\rangle(\sin(2\theta)|H\rangle - \cos(2\theta)|V\rangle))/\sqrt{2}$ (see the Supplemental Material [18] for details). In order to achieve the intended four-qubit state, coupling to the path degree of freedom is required. Thus, the polarization dependence of the output of a polarizing beam splitter is used; i.e., photons are transformed as $|H\rangle \to |Ha\rangle$ and $|V\rangle \to |Vb\rangle$ with a and b denoting the corresponding output modes of the PBS, see Fig. 2(b). Consequently, four-qubit states parametrized by θ and ϕ, $|\Psi(\theta,\phi)\rangle = (\cos(2\theta)|HaHa\rangle + \sin(2\theta)|HaVb\rangle + e^{i\phi}\sin(2\theta)|VbHa\rangle - e^{i\phi}\cos(2\theta)|VbVb\rangle)/\sqrt{2}$, are obtained. Prominent members of $|\Psi(\theta,\phi)\rangle$ are for example the GHZ states $(|HaHa\rangle \mp |VbVb\rangle)/\sqrt{2}$ for $\theta = 0$ and $\phi = 0, \pi$, respectively, or the cluster states $(|HaHa\rangle + |HaVb\rangle \pm |VbHa\rangle \mp |VbVb\rangle)/2$ obtained fo r $\theta = \pi/8$ and $\phi = 0, \pi$.

The prepared states are characterized by means of QST, proving full control of the experimental apparatus. This can be achieved with an interferometer setup as shown in Fig. 2(b), overlapping the modes a and b together with a polarization analysis and coincidence detection in the outputs.

Experimental results.—Thirteen states were prepared with $\phi = \pi$ and θ being increased from 0 (GHZ) to $\pi/8$ (cluster) and to $\pi/4$ (GHZ′) in equidistant steps. The coincidence rate was approximately $100\ \mathrm{s}^{-1}$ with a measurement time of 40 s for each basis setting, resulting in 3700–4400 counts per setting and a measurement time of about 12 h to perform QST for all states. A measure for the quality of a prepared state ϱ_{exp} with respect to a pure target state $|\psi\rangle$ is the fidelity $\mathcal{F} = \mathrm{Tr}(\varrho_{\mathrm{exp}}|\psi\rangle\langle\psi|)$. For the GHZ state, we observed a fidelity of $\mathcal{F} = 0.958 \pm 0.004$, while for the cluster state it was $\mathcal{F} = 0.962 \pm 0.003$. For the other states, see Table IV in the Supplemental Material [18].

Genuine four-partite entanglement could be tested using two measurement settings only. Let us start to determine the witnesses for the GHZ state from measuring two settings $\mathcal{M}_{3333}$ and $\mathcal{M}_{1221}$. The values of the respective measured correlations (Table III in the Supplemental Material [18]) lead to a violation of all seven criteria by at least 56 standard deviations for all cuts, see Table II. Also, the combined criterion $\mathcal{W}^{\mathrm{GHZ}} = 0.916 \pm 0.005 > \frac{7}{11}$ certifies genuine four-partite entanglement. For the cluster state,

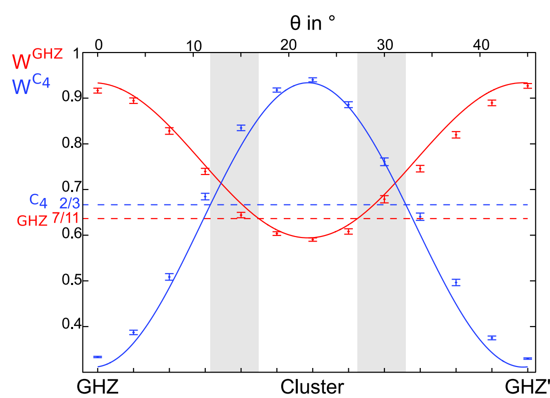

FIG. 3. The entanglement criterion for the GHZ states allows us to detect most of the superpositions of GHZ and the cluster state to be genuinely four-partite entangled (red, starting at 0.92) whereas the criterion for the cluster state detects states around $\theta = 22.5°$ to be genuinely four-partite entangled (blue, starting at 0.33). States within the gray shaded areas can be detected to be genuinely four-partite entangled by means of both criteria. The solid lines show the theoretically expected values for the target states $|\Psi(\theta,\phi)\rangle$ mixed with white noise such that on average the fidelities correspond to the measured values.

according to our entanglement criterion, the measurement settings $\mathcal{M}_{1133}$ and $\mathcal{M}_{3311}$ were used (see Supplemental Material [18]), resulting in $\mathcal{W}^{\mathcal{C}_4} = 0.940 \pm 0.004 > \frac{2}{3}$ for the combined criterion.

Using the combined witnesses, we analyze the entanglement for all states $|\Psi(\theta,\phi)\rangle$ (Fig. 3). As can be seen, 10 of 13 states can be detected as genuinely four-partite entangled by the criterion $\mathcal{W}^{\mathrm{GHZ}}$, the 6 states close to the cluster state can be determined by means of $\mathcal{W}^{\mathcal{C}_4}$. Some states can be shown to be truly four-partite entangled by means of both criteria as both are above their respective threshold. Genuine four-partite entanglement could be proven with experimental data of the Dicke state $|D_4^{(2)}\rangle$ [24] and the singlet state [25], see Table II. For more details see the Supplemental Material [18].

Conclusion.—We have introduced a novel scheme for the systematic construction of entanglement witnesses, which need a minimal number of measurements for their evaluation independent of the number of qubits. We believe that such a minimal multipartite entanglement detection will become a handy diagnostic procedure as it is fast and simple. An interesting question is what other states can reveal their multipartite quantum correlations in two measurements. Another challenge is to find even stronger criteria, which, by possibly going to few more measurements, will detect multipartite entanglement with a higher robustness against noise.

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