Genuine Multipartite Entanglement without Multipartite Correlations

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Nonclassical correlations between measurement results make entanglement the essence of quantum physics and the main resource for quantum information applications. Surprisingly, there are n-particle states which do not exhibit n-partite correlations at all but still are genuinely n-partite entangled. We introduce a general construction principle for such states, implement them in a multiphoton experiment and analyze their properties in detail. Remarkably, even without multipartite correlations, these states do violate Bell inequalities showing that there is no classical, i.e., local realistic model describing their properties.

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Correlations between measurement results are the most prominent feature of entanglement. They made Einstein, Podolski, and Rosen [1] question the completeness of quantum mechanics and are nowadays the main ingredient for the many applications of quantum information like entanglement based quantum key distribution [2] or quantum teleportation [3].

Correlations enable us, e.g., when observing two maximally entangled qubits, to use a measurement result observed on the first system to infer exactly the measurement result on the second system. In this scenario, the two particle correlations are formally given by the expectation value of the product of the measurement results obtained by the two observers. Note, the single particle correlation, i.e., the expectation value of the results for one or the other particle are zero in this case. Consequently, we cannot predict anything about the individual results. When studying the entanglement between n particles, a natural extension is to consider n-partite correlations, i.e., the expectation value of the product of n measurement results. Such correlation functions are frequently used in classical statistics and signal analysis [4], moreover, in quantum information, almost all standard tools for analyzing multipartite systems like multiparty entanglement witnesses [5,6] and Bell inequalities [7,8] are based on the n-partite correlation functions.

Recently, Kaszlikowski *et al.* [9] pointed at a particular quantum state with vanishing multiparty correlations which, however, is genuinely multipartite entangled. This discovery, of course, prompted vivid discussions on a viable definition of classical and quantum correlations [10,11]. Still, the question remains what makes up such states with no full n-partite correlations and how nonclassical they can be, i.e., whether they are not only entangled but whether they also violate a Bell inequality.

Here, we generalize, highlight, and experimentally test such remarkable quantum states. We introduce a simple principle how to construct states without n-partite correlations for odd n and show that there are infinitely many such states which are genuinely n-partite entangled. We implement three and five qubit no-correlation states in a multiphoton experiment and demonstrate that these states do not exhibit n-partite correlations. Yet, due to the existence of correlations between a smaller number of particles, we observe genuine n-partite entanglement. Using our recently developed method to design n-partite Bell inequalities from lower order correlation functions only [12,13], we show that these states, despite not having full correlations, can violate Bell inequalities.

Correlations.—The quantum mechanical correlation function $T_{j_1 \dots j_n}$ is defined as the expectation value of the product of the results of n observers

$$T_{j_1 \dots j_n} = \langle r_1 \dots r_n \rangle = \mathrm{Tr}(\rho \sigma_{j_1} \otimes \dots \otimes \sigma_{j_n}), \qquad (1)$$

where r_k is the outcome of the local measurement of the kth observer, parametrized by the Pauli operator σ_{j_k} with $j_k \in \{x, y, z\}$. Evidently, besides the n-partite correlations, for an n-partite state, one can also define $l < n$ fold correlations $T_{\mu_1 \dots \mu_n} = \mathrm{Tr}(\rho \sigma_{\mu_1} \otimes \dots \otimes \sigma_{\mu_n})$ with $\mu_i \in \{0, x, y, z\}$ and $|\{\mu_i = 0\}| = n - l$. Nonvanishing l-fold correlations indicate that we can infer (with higher probability of success than pure guessing) an lth measurement result from the *product* of the other $(l-1)$ results [see Supplemental Material [14]]. Only in the two particle scenario can we directly use the result from one measurement to infer the other result. For an n-qubit no-correlation state, the vanishing n-partite correlations do not imply vanishing correlations between a smaller number of observers, thus not necessarily destroying predictability. We will see also in the experimentally implemented example that the various individual results still enable some possibility for inference, which is then largely due to bipartite correlations.

Constructing no-correlation states.—For any state $|\psi\rangle$ with an odd number n of qubits, we can construct an "antistate" $|\bar{\psi}\rangle$, i.e., the state whose n-partite correlations are inverted with respect to the initial one. By evenly mixing these states

$$\rho_\psi^{nc} = \frac{1}{2}|\psi\rangle\langle\psi| + \frac{1}{2}|\bar{\psi}\rangle\langle\bar{\psi}|, \qquad (2)$$

we obtain a state ρ_ψ^{nc} without n-partite correlations.

The antistate $|\bar{\psi}\rangle$ of a state $|\psi\rangle$ described in the computational basis by

$$|\psi\rangle = \sum_{k_1,\ldots,k_n=0}^{1} \alpha_{k_1,\ldots,k_n}|k_1\ldots k_n\rangle, \qquad (3)$$

with normalized coefficients $\alpha_{k_1,\ldots,k_n} \in \mathbb{C}$, is given by

$$|\bar{\psi}\rangle \equiv \sum_{k_1,\ldots,k_n=0}^{1} (-1)^{k_1+\cdots+k_n}\alpha^*_{1-k_1,\ldots,1-k_n}|k_1\ldots k_n\rangle, \qquad (4)$$

where the asterisk denotes complex conjugation. This state has inverted correlations with respect to those in $|\psi\rangle$ for every odd number of observers, whereas all the correlation function values for an even number of observers remain unchanged.

$|\bar{\psi}\rangle$ is mathematically obtained from $|\psi\rangle$ by applying local universal-not gates [24]. These gates introduce a minus sign to all local Pauli operators. Therefore, for odd n, the correlations of $|\bar{\psi}\rangle$ have opposite sign to those of $|\psi\rangle$. Representing the universal-not gate by $N = \sigma_z\sigma_x K$, where K is the complex conjugation operating in the computational basis, i.e., $K(\alpha|0\rangle + \beta|1\rangle) = \alpha^*|0\rangle + \beta^*|1\rangle$, indeed, we obtain $N\sigma_x N^\dagger = -\sigma_x$, $N\sigma_y N^\dagger = -\sigma_y$, and $N\sigma_z N^\dagger = -\sigma_z$. Applying N to all the n subsystems, we find the anticipated result $N \otimes \cdots \otimes N|\psi\rangle = |\bar{\psi}\rangle$.

Although N is antiunitary, $|\bar{\psi}\rangle$ is always a proper physical state and can be obtained by some global transformation of $|\psi\rangle$. In general, N can be approximated [25], but if all the coefficients $\alpha_{k_1\ldots k_n}$ are real, complex conjugation can be omitted and no-correlation states can be generated by local operations.

This construction principle can be generalized to mixed states using $\bar{\rho} = N^{\otimes n}\rho(N^{\otimes n})^\dagger$, which changes every pure state in the spectral form to the respective antistate. Evenly mixing ρ and $\bar{\rho}$ therefore produces a state with no l-party correlations for all odd l.

One may then wonder whether the principle of Eq. (2) can also be applied to construct a no-correlation state for every state with an even number of qubits. The answer is negative as shown by the following counterexample. Consider the Greenberger-Horne-Zeilinger state of an even number of qubits $|\psi\rangle = (1/\sqrt{2})(|0\ldots0\rangle + |1\ldots1\rangle)$. It has nonvanishing $T_{z\ldots z}$, 2^{n-1} multipartite correlations in the xy plane, and also, $2^{n-1} - 1$ correlations between a smaller number of subsystems, all equal to ±1. However, for a state with inverted correlations between all n parties (making no assumptions about the correlations between smaller numbers of observers), the fidelity relative to the GHZ state, given by $\frac{1}{2^n}\sum_{\mu_1,\ldots,\mu_n=0}^{3} T^{\mathrm{GHZ}}_{\mu_1\ldots\mu_n} T^{\mathrm{anti}}_{\mu_1\ldots\mu_n}$, is negative because more than half of the correlations are opposite. Hence, this state is unphysical and there is no such "antistate". In fact, so far we were unable to find an antistate to *any* genuinely multiqubit entangled state of even n.

Entanglement without correlations: infinite family.—Consider a three-qubit system in the pure state

$$|\phi\rangle = \sin\beta\cos\alpha|001\rangle + \sin\beta\sin\alpha|010\rangle + \cos\beta|100\rangle, \qquad (5)$$

where $\alpha, \beta \in (0, \pi/2)$ (which includes the state $|W\rangle$ with $\alpha = \pi/3$ and $\beta = \cos^{-1}(1/\sqrt{3})$). Together with any local unitary transformation thereof, this defines a three dimensional subspace of genuinely tripartite entangled states within the eight dimensional space of three qubit states. To show that all the respective no-correlation states ρ_ϕ^{nc} are genuinely entangled, we use a criterion similar to the one in [6], i.e.,

$$\max_{T^{\text{bi-prod}}}(T, T^{\text{bi-prod}}) < (T, T^{\exp}) \Rightarrow \rho^{\exp} \text{ is not biseparable}, \qquad (6)$$

where maximization is over all biproduct pure states and $(U, V) \equiv \sum_{\mu,\nu,\eta=0}^{3} U_{\mu\nu\eta}V_{\mu\nu\eta}$ denotes the inner product in the vector space of correlation tensors. Condition [Eq. (6)] can be interpreted as an entanglement witness $\mathcal{W} = \alpha\mathbb{1} - \rho_\phi^{nc}$, where $\alpha = L/8$ and $L = \max_{T^{\text{bi-prod}}}(T, T^{\text{biproduct}})$ is the left-hand side of Eq. (6). In the ideal case of preparing $\rho^{\exp}$ perfectly, $T^{\exp} = T$, the right-hand side of our criterion equals four for all the states of the family, and thus, the expectation value of the witness is given by $\mathrm{Tr}(\mathcal{W}\rho_\phi^{nc}) = (L - 4)/8$.

A simple argument for ρ_ϕ^{nc} being genuinely tripartite entangled can be obtained from the observation that $|\phi\rangle$ and $|\bar{\phi}\rangle$ span a two-dimensional subspace of the three qubit Hilbert space [9]. As none of the states $|\Phi\rangle = a|\phi\rangle + b|\bar{\phi}\rangle$ is a biproduct (for the proof see Supplemental Material [14]), states in their convex hull do not intersect with the subspace of biseparable states and thus all its states, including ρ_ϕ^{nc} are genuinely tripartite entangled. To evaluate the entanglement in the experiment, we calculated L for all states of Eq. (5). We obtain $L_{|\phi\rangle} < 4$ in general, with $L_{|W\rangle} = 10/3$. Similar techniques were used to analyze five-qubit systems.

Quantum correlations without classical correlations?—The cumulants and correlations were initially proposed as a measure of genuinely multiparty nonclassicality in Ref. [26]. Kaszlikowski *et al.* [9], however, showed that such a quantification is not sufficient as the state ρ_W^{nc} has vanishing cumulants, yet contains genuinely multiparty entanglement. They suggested that the vanishing cumulants or standard correlation functions [Eq. (1)] indicate the lack of genuine multiparty "classical" correlations. This initiated a vivid discussion on a proper definition and measure of genuine multipartite "classical" and quantum correlations. Bennett *et al.* proposed a set of axioms for measures of genuine multipartite correlations [11]. They showed that the correlation function [Eq. (1)] does not fulfill all the requirements, but also still strive for computable measures that satisfy these axioms [15,27]. An information-theoretic definition of multipartite correlations was given by Giorgi *et al.* [15]. Their measure combines the entropy of all sizes of subsystems. Applying their definitions to ρ_W^{nc}, we obtain genuine classical tripartite correlations of 0.813 bit and genuine quantum tripartite correlations of 0.439 bit resulting in total genuine tripartite correlations of 1.252 bit (see Supplemental Material [14] for calculations for all ρ_ϕ^{nc}). While this approach does assign classical correlations in the context of Giorgi *et al.* [15] to ρ_W^{nc}, it does not fulfill all requirements of [11] either.

Experiment.—The three photon state $|W\rangle$ can be observed either using a multiphoton interferometer setup [28] or by suitably projecting the fourth photon of a 4-photon symmetric Dicke state [29]. The latter scheme has the advantage that it also offers the option to prepare the states $|\bar{W}\rangle$ and ρ_W^{nc}. The states $|W\rangle$ and $|\bar{W}\rangle$ are particular representatives of the symmetric Dicke states, which are defined as

$$|D_n^{(e)}\rangle = \binom{n}{e}^{-1/2}\sum_i \mathcal{P}_i(|H^{\otimes(n-e)}\rangle \otimes |V^{\otimes e}\rangle), \quad (7)$$

where $|H/V\rangle$ denotes horizontal (vertical) polarization and $\mathcal{P}_i$ all distinct permutations, and with the three photon states $|W\rangle = |D_3^{(1)}\rangle$ and $|\bar{W}\rangle = |D_3^{(2)}\rangle$. We observed four- and six-photon Dicke states using a pulsed collinear type II spontaneous parametric down conversion source together with a linear optical setup (see Fig. 1) [30,31]. The $|D_n^{(e)}\rangle$ states were observed upon detection of one photon in each of the four or six spatial modes, respectively. We characterized the state $|D_4^{(2)}\rangle$ by means of quantum state tomography, i.e., a polarization analysis in each mode, collecting for each setting 26 minutes of data at a rate of 70 events per minute. The fidelity of the experimental state $|D_4^{(2)}\rangle^{\mathrm{exp}}$ was directly determined from the observed frequencies together with Gaussian error propagation as 0.920 ± 0.005, which due to the high number of detected events [16] is compatible with the value 0.917 ± 0.002 as obtained from a maximum likelihood (ML) reconstruction and nonparametric bootstrapping [14,20]. The high quality achieved here allowed a precise study of the respective states. The fidelities of the observed three qubit states with respect to their target states are 0.939 ± 0.011 for $|W\rangle^{\mathrm{exp}}$, 0.919 ± 0.010 for $|\bar{W}\rangle^{\mathrm{exp}}$, and 0.961 ± 0.003 for $\rho_W^{nc,\mathrm{exp}}$. Analogously, starting with a six-photon Dicke state $|D_6^{(3)}\rangle$

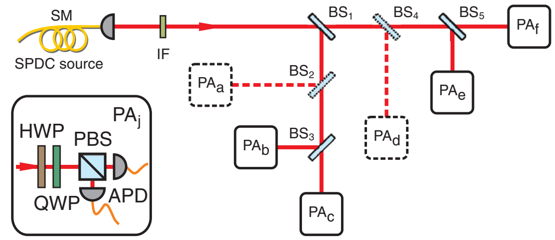

FIG. 1 (color online). Schematic of the linear optical setup used to observe symmetric Dicke states from which states with vanishing 3- and 5-partite correlations can be obtained. The photons are created by means of a cavity enhanced pulsed collinear type II spontaneous parametric down conversion source pumped at 390 nm [31]. Distributing the photons symmetrically into six modes by five beam splitters (BS) enables the observation of the state $|D_6^{(3)}\rangle$. Removing beam splitters BS_2 and BS_4 reduces the number of modes to four and thus the state $|D_4^{(2)}\rangle$ is obtained. State analysis is enabled by sets of half wave (HWP) and quarter-wave plates (QWP) together with polarizing beam splitters (PBS) in each mode. The photons are measured by fiber-coupled single photon counting modules connected to a coincidence logic [30].

[32], we could also analyze the properties of the five photon state $\rho_{D_5^{(2)}}^{nc}$. The five-qubit fidelity of $\rho_{D_5^{(2)}}^{nc,\mathrm{exp}}$ is determined via a ML reconstruction from fivefold coincidences to be 0.911 ± 0.004 (for the detailed characterization see Supplemental Material [14]).

For the experimental analysis of the states, we start by determining T_{zzz} for the three states $|W\rangle^{\mathrm{exp}}$, $|\bar{W}\rangle^{\mathrm{exp}}$, and $\rho_W^{nc,\mathrm{exp}}$. As the first two have complementary structure of detection probabilities (with $T_{zzz} = -0.914 \pm 0.034$ and $T_{zzz} = 0.904 \pm 0.034$, respectively), weighted mixing of these states leads to $\rho_W^{nc,\mathrm{exp}}$ with $T_{zzz} = 0.022 \pm 0.023$, i.e., a correlation value compatible with 0 (see Supplemental Material [14]). Figure 2 presents experimental data for all possible tripartite correlations of the observed states. Assuming a normal distribution centered at zero with a standard deviation given by our experimental errors, the observed correlations have a p value of 0.44 for the Anderson-Darling test, which shows that indeed one can adhere to the hypothesis of vanishing full correlations. Similarly, the five qubit state $\rho_{D_5^{(2)}}^{nc,\mathrm{exp}}$ exhibits strongly suppressed, almost vanishing correlations. For details on the five qubit state, please see Supplemental Material [14].

We want to emphasize that the vanishing tripartite correlations of $\rho_W^{nc,\mathrm{exp}}$ are no artifact of measuring in the Pauli bases. In fact, all states obtained via local unitary transformations do not exhibit any n-partite correlations. To illustrate this property, we considered correlation measurements in non-standard bases. As an example, we chose measurements in the zy plane $\sigma_\theta = \cos\theta\sigma_z + \sin\theta\sigma_y$ with $\theta \in [0, 2\pi]$ ($\sigma_\phi = \cos\phi\sigma_y + \sin\phi\sigma_z$ with $\phi \in [0, 2\pi]$) for the first (second) qubit resulting in the correlations $T_{\theta j_2 j_3} = \mathrm{Tr}(\rho\sigma_\theta \otimes \sigma_{j_2} \otimes \sigma_{j_3})$ ($T_{j_1\phi j_3}$). Indeed, as shown in

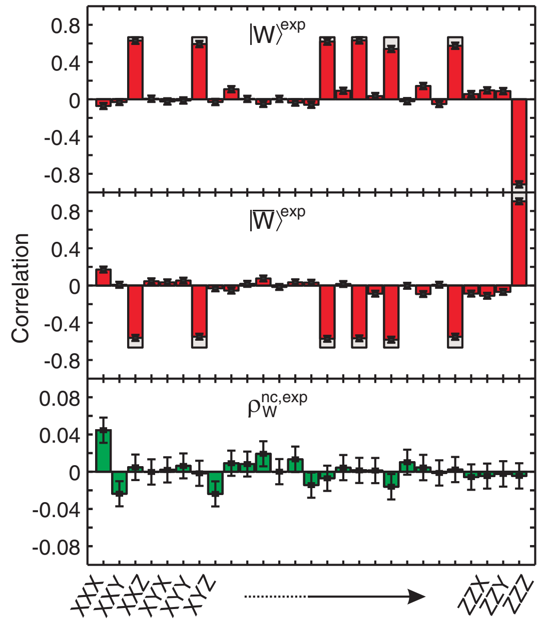

FIG. 2 (color online). Experimental tripartite correlations (red) for $|W\rangle^{\rm exp}$, $|\bar{W}\rangle^{\rm exp}$, and (green) $\rho_W^{nc,\rm exp}$ in comparison to the theoretically expected values (gray). Note that the correlations of the state $\rho_W^{nc,\rm exp}$ are magnified by a factor of 10. The plot presents measured values of $T_{j_1 j_2 j_3}$ for the observables listed below the plot. Obviously, the states $|W\rangle^{\rm exp}$ and $|\bar{W}\rangle^{\rm exp}$ have opposite tripartite correlations canceling each other when they are mixed.

Fig. 3, $T_{\theta j_2 j_3}$ ($T_{j_1 \phi j_3}$) vanishes independently of the choice of θ (ϕ). In contrast, the bipartite correlations $T_{\theta z0}$ ($T_{y\phi 0}$) between qubit 1 and 2 do not vanish at all and clearly depend on θ (ϕ). By means of those even number correlations, one is still able to infer the result of another party from ones own result with probability $2/3 > 1/2$. For example, the values of $T_{zz0} = -1/3$ ($T_{z0z} = -1/3$) indicate that knowing, e.g., result "0" for the first qubit, we can infer that the result will be "1" with $p = 2/3$ on the second (third) qubit, etc.

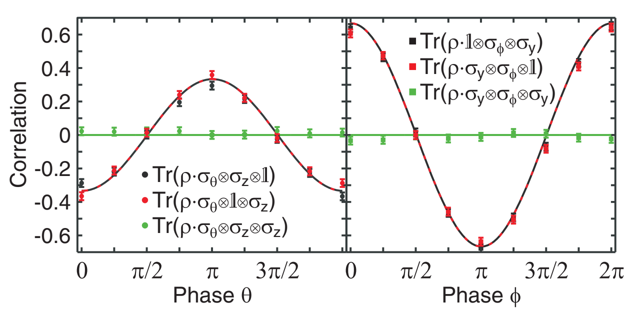

FIG. 3 (color online). Vanishing tripartite correlations for arbitrary measurements and non-vanishing bipartite correlations. Observable σ_θ (σ_ϕ) was measured on the first (second) qubit and σ_z (σ_y) measurements were performed on both other qubits (green curves) or one of them (red and black curves). The solid lines show the theoretically expected curves.

Although the three qubits are not tripartite correlated, the *bi*partite correlations shown above give rise to genuine *tri*partite entanglement. This can be tested for the experimental states employing Eq. (6). We observe

$$(T, T_W^{nc,\rm exp}) = 3.858 \pm 0.079 > 3.33\bar{3},$$
$$(T, T_{D_5^{(2)}}^{nc,\rm exp}) = 13.663 \pm 0.340 > 12.8,$$

both above the respective biseparable bound of $10/3 = 3.33\bar{3}$ (12.8) by more than 6.6 (2.4) standard deviations, proving that in spite of vanishing full correlations the states are genuinely tripartite (five-partite) entangled [14].

The observed five-photon state has one more remarkable property [13]. For this state, every correlation between a fixed number of observers, i.e., bipartite correlations, tripartite correlations, etc. admits description with an explicit local hidden-variable model [8]. However, some of the models are different and thus cannot be combined in a single one. Using linear programming to find joint probability distributions reproducing quantum predictions [12], we obtain an optimal Bell inequality using only two- and four-partite correlations [13]. From the observed data, we evaluate the Bell parameter to be $\mathcal{B} = 6.358 \pm 0.149$ which violates the local realistic bound of 6 by 2.4 standard deviations [33]. This violation confirms the nonclassicality [14] of this no-correlation state and also offers its applicability for quantum communication complexity tasks. Contrary to previous schemes, here, the communication problem can be solved in every instance already by only a subset of the communicating parties [35].

Conclusions.—We introduced a systematic way to define and to experimentally observe mixed multipartite states with no n-partite correlations for odd n, as measured by standard correlation functions. For the first time, we experimentally observed a state which allowed the violation of a Bell inequality without full correlations, thereby proving both the nonclassicality of no-correlation states as well as their applicability for quantum communication protocols. The remarkable properties of these states prompt intriguing questions. For example, what might be the dimensionality of these states or their respective subspaces, or whether we can even extend the subspace of states and antistates which give genuinely entangled no-correlation states? Moreover, can no-correlation states be used for quantum protocols beyond communication complexity, and, of course, whether these remarkable features can be cast into rigorous and easily calculable measures of genuine correlations satisfying natural postulates [11]?

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